Obstruction to small-time local controllability on a KdV control system

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New blow-up rates for fast controls of Schrödinger and heat equations

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FAST AND STRONGLY LOCALIZED OBSERVATION FOR THE SCHRÖDINGER EQUATION

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SINGLE INPUT CONTROLLABILITY OF A SIMPLIFIED FLUID-STRUCTURE INTERACTION MODEL

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Andreas Hartmann · Karim Kellay · Marius Tucsnak

From the reachable space of the heat equation to Hilbert spaces of holomorphic functions

Marius Tucsnak
George Weiss

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A KdV control system with only one control

Following L. Rosier (1997), we consider the KdV control system

$$
(1) \qquad \begin{cases} y_t + y_x + y_{xxx} + yy_x = 0, \; t \in [0,T], \; x \in [0,L], \\ y(t,0) = y(t,L) = 0, \; y_x(t,L) = u(t), \; t \in [0,T], \end{cases}
$$

where, at time $t \in [0,T]$, the state is $y(t,\cdot) \in L^2(0,L)$ and the control is $u(t) \in \mathbb{R}$.

Controllability of the linearized control system

The linearized control system (around 0) is

$$(2) \qquad \begin{cases} y_t + y_x + y_{xxx} = 0,\ t \in [0,T],\ x \in [0,L], \\ y(t,0) = y(t,L) = 0,\ y_x(t,L) = u(t),\ t \in [0,T], \end{cases}$$

where, at time $t \in [0,T]$, the state is $y(t,\cdot) \in L^2(0,L)$ and the control is $u(t) \in \mathbb{R}$.

Theorem (L. Rosier (1997))

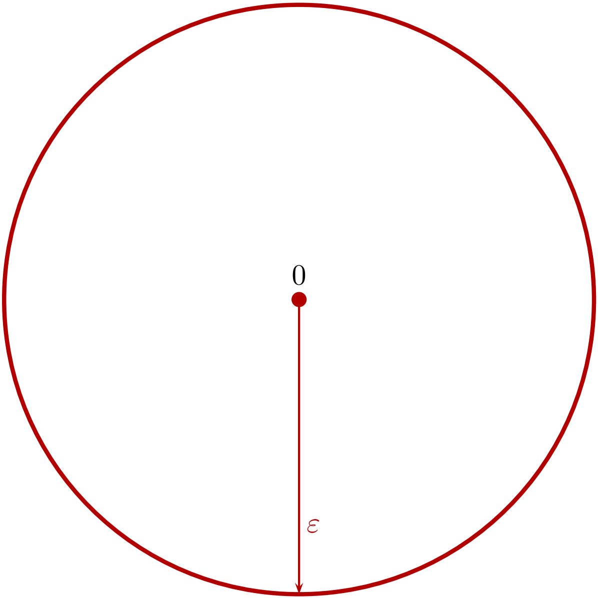
For every $T > 0$, the linearized control system is controllable in time T if and only

$$(3) \qquad L \notin \mathcal{N} := \left\{ 2\pi \sqrt{\frac{k^2 + kl + l^2}{3}},\ k \in \mathbb{N}^*,\ l \in \mathbb{N}^* \right\}.$$

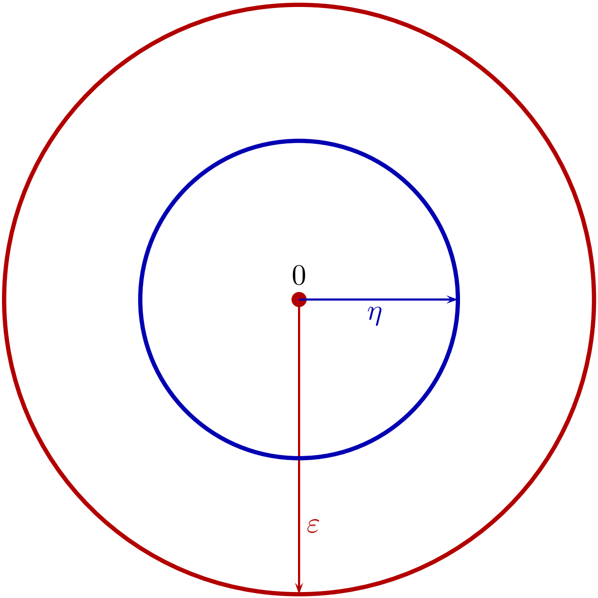
Moreover, if $L \in \mathcal{N}$, the uncontrollable part is a linear space (later denoted $\mathcal{M}$) of finite dimension.

Local controllability in time T

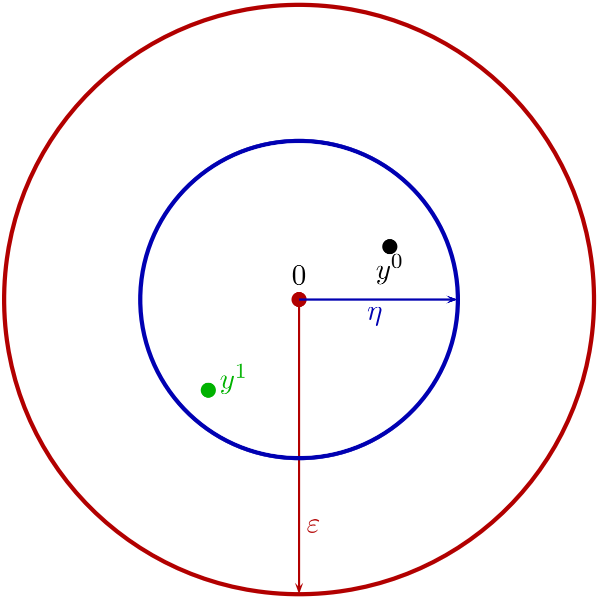
Local controllability in time T



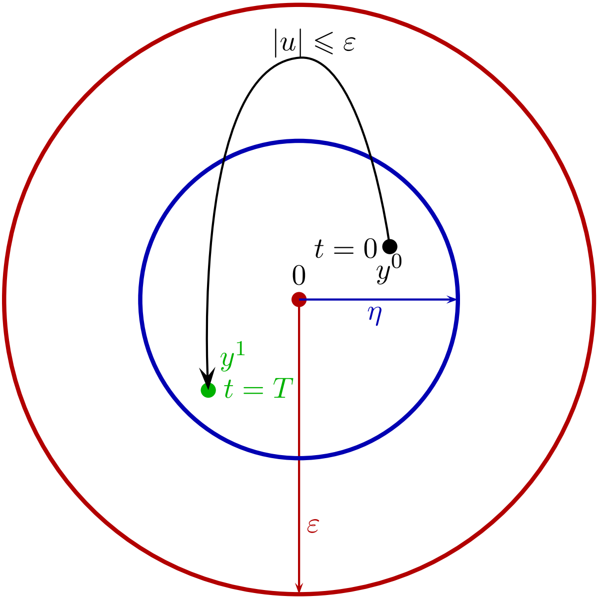
Local controllability in time T



Local controllability in time T



Local controllability in time T



Application to the nonlinear system

Theorem (L. Rosier (1997))

For every $T > 0$, the KdV control system is locally controllable (around 0) in time T if $L \notin \mathcal{N}$ for the L^2-norm for the state and the L^2-norm for the control.

Remark

The above controllability property is called Small-Time Local Controllability, STLC in short: the time, the state, and the controls are small (for suitable norms).

Controllability when $L \in \mathcal{N}$

Theorem (STLC if $\dim(\mathcal{M}) = 1$, JMC and E. Crépeau (2004))

If the uncontrollable part $\mathcal{M}$ of the linearized system is of dimension 1, for every $T > 0$ the KdV control system is locally controllable (around 0) in time T.

Remark

If $L = 2\pi$, $\mathcal{M}$ is of dimension 1 and there are infinitely many L such that $\mathcal{M}$ is of dimension 1.

Theorem (Local controllability in large time, E. Cerpa (2007), E. Cerpa and E. Crépeau (2008))

For every $L \in \mathcal{N}$, there exists $T > 0$ such that the KdV control system is locally controllable (around 0) in time T.

The proofs of these theorems rely on the power series expansion method. In the first theorem an expansion to the order 3 is required, while in the secund theorem an expansion to the order 2 is used. For the order 3 the computations are more complicate but the fact that this order is odd helps to get the local controllability in small time.

STLC for every $L \in \mathcal{N}$?

Question (Small-time local controllability)

Assume that dim$(\mathcal{M}) > 1$. *Is is true that for every $T > 0$ the control system*

(4)
$$\begin{cases} y_t + y_x + y_{xxx} + yy_x = 0,\ t \in [0,T],\ x \in [0,L], \\ y(t,0) = y(t,L) = 0,\ y_x(t,L) = u(t),\ t \in [0,T], \end{cases}$$

is locally controllable in time T?

Theorem (JMC, A. Koenig and H.-M. Nguyen (2020))

Let $k,\, l \in \mathbb{N}\setminus\{0\}$ *be such that* $2k+l \notin 3\mathbb{N}$. *Assume that*

(5) $$L = 2\pi\sqrt{\frac{k^2+kl+l^2}{3}}.$$

Then our KdV control system is not small-time locally null-controllable with controls in H^1 *and initial datum in* $H^3(0,L)\cap H^1_0(0,L)$, *i.e., there exist* $T_0>0$ *and* $\varepsilon_0>0$ *such that, for every* $\delta>0$, *there is* $y_0\in H^3(0,L)\cap H^1_0(0,L)$ *with* $\|y_0\|_{H^3(0,L)}<\delta$ *such that for every* $u\in H^1(0,T_0)$ *with* $\|u\|_{H^1(0,T_0)}<\varepsilon_0$ *and* $u(0)=y_0'(L)$, *we have*

(6) $$y(T_0,\cdot)\neq 0,$$

where $y\in C\big([0,T_0];H^3(0,L)\big)\cap L^2\big([0,T_0];H^4(0,L)\big)$ *is the unique solution of our control system for the control* u *and starting from* y_0.

Open problem (Regularity and small-time local controllability)

Is the KdV control system is small-time locally null controllable with initial in $L^2(0,L)$ and control in $L^2(0,T)$ for a critical length as in the previous theorem?

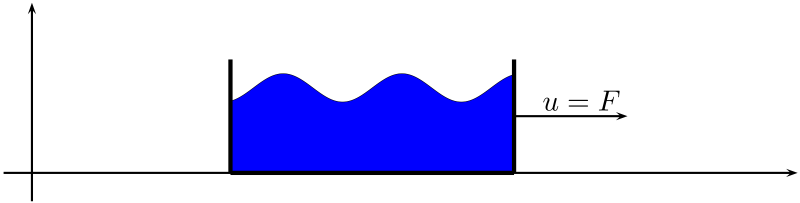
Open problem (Just $\dim \mathcal{M} > 1$)

Can the assumption $2k + l \notin 3\mathbb{N}$ be replaced by the weaker assumption $\dim \mathcal{M} > 1$?

Open problem (Optimal time)

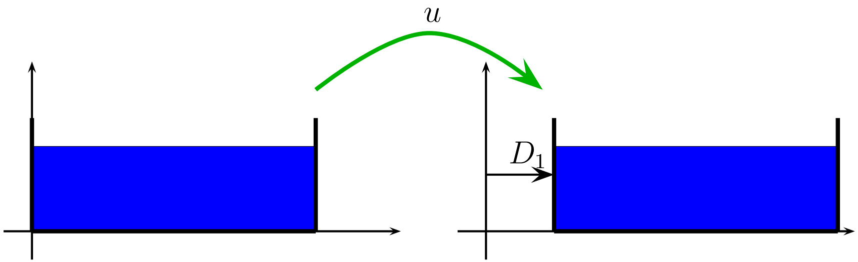
What is the minimal time for local controllability?

Another obstruction: The water tank control system



The modelling is done with the Saint-Venant equations. See F. Dubois, N. Petit and P. Rouchon (1999).

Steady-state controllability



F. Dubois, N. Petit and P. Rouchon proved in 1999 that for the linearized system around the equilibrium $H = H_e$, speed=0, position of the tank at the origin, the steady-state controllability is valid for every time T such that

$$T > \sqrt{\frac{LH_e}{g}}. \tag{7}$$

However we have the following theorem

Theorem (JMC, A. Koenig and H.-M. Nguyen (2021))

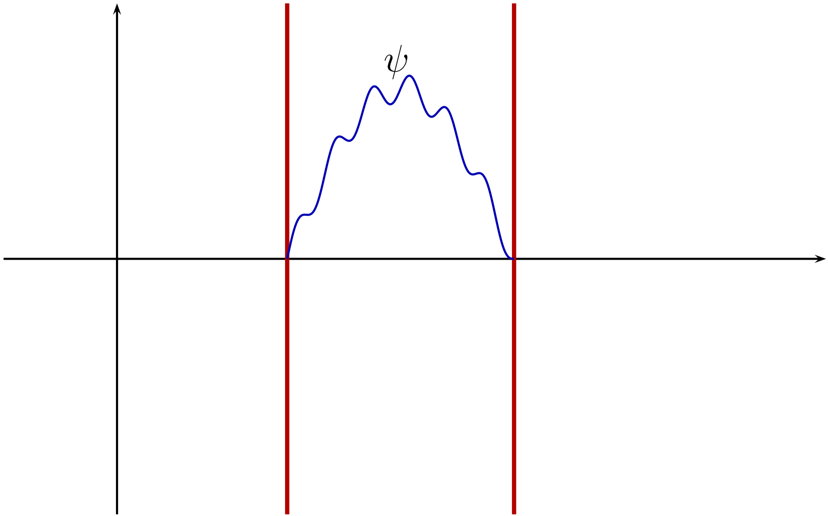
For every $T < 2\sqrt{\frac{LH_e}{g}}$ the steady-state controllability with small (for the $H^{3/2}$-norm) control does not hold in time T even if the two steady states are arbitrary close but different.

Remark

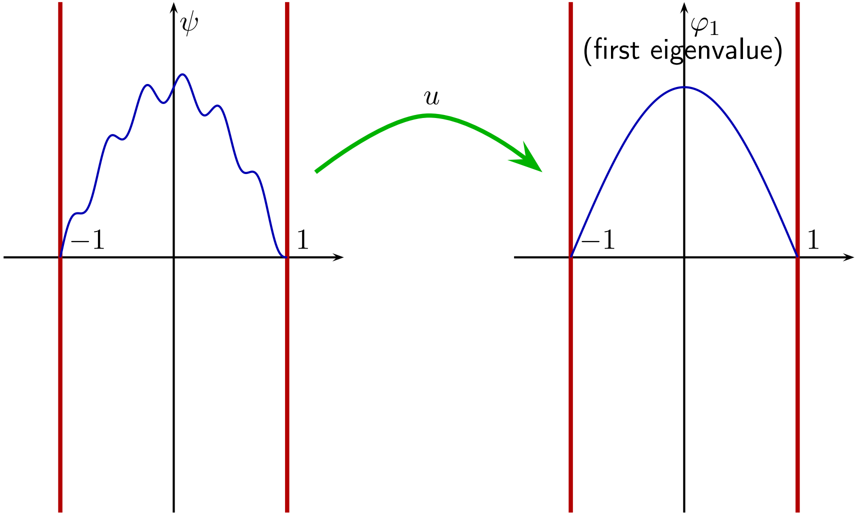
The steady-state controllability holds for large enough time (JMC (2002)).

A quantum particle in a moving box

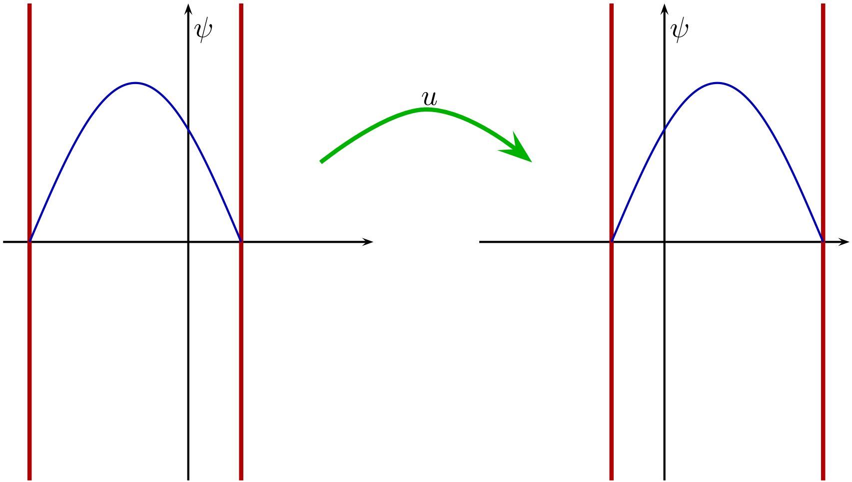
(Suggested by P. Rouchon)



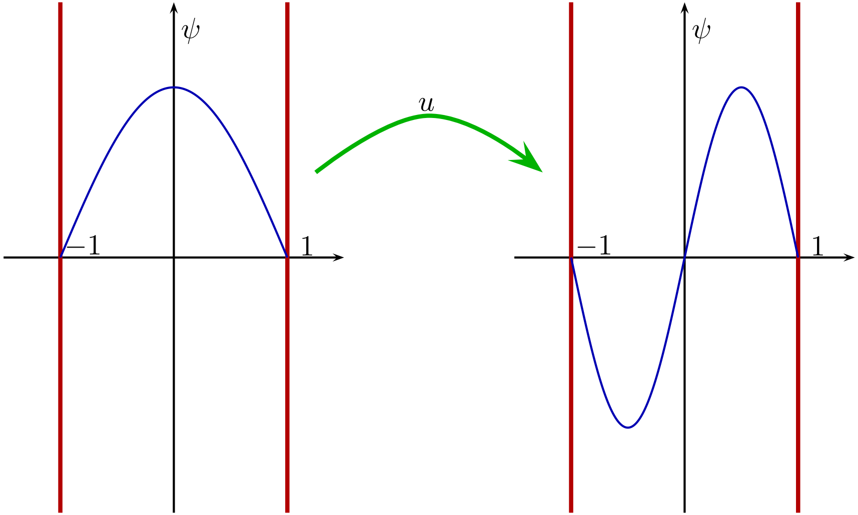
Local ("null") controllability



Motion of the box



Global controllability: From the first eigenfunction to the second one

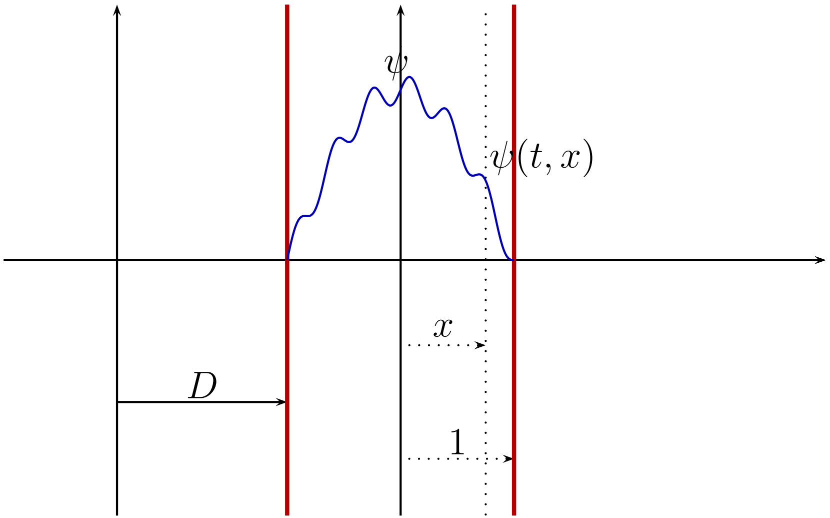


Controllability results

Theorem

- *The steady-state motion of the box for the linearized control system around the first eigenfunction holds in small time: P. Rouchon (2003). However this result does not hold for the (nonlinear) system JMC (2006) for small controls and arbitrary small but not 0 displacement.*
- *Large time local controllability: Without* (S, D)*: K. Beauchard (2005); with* (S, D)*: K. Beauchard and JMC (2006),*
- *Large time controllability between eigenfunctions: K. Beauchard and JMC (2006),*
- *Large time global controllability: V. Nersesyan (2008).*

Not STLC: Notations



Not STLC: Equations and definitions

Let $\varepsilon \in (0, 1]$. Let $u : (0, T) \to \mathbb{R}$ be such that

$$|u(t)| < \varepsilon,\ t \in (0, T). \tag{8}$$

Let $\psi_1(t, x) = e^{i\lambda_1 t}\varphi_1(x)$. Let (ψ, S, D) be the solution of the Cauchy problem (the control system (P. Rouchon))

$$i\psi_t = -\psi_{xx} - u(t)x\psi,\ (t, x) \in (0, T) \times (-1, 1), \tag{9}$$

$$\psi(t, -1) = \psi(t, 1) = 0,\ t \in (0, T), \tag{10}$$

$$\dot{S}(t) = u(t),\ \dot{D}(t) = S(t),\ t \in (0, T), \tag{11}$$

$$\psi(0, x) = \psi_1(0, x),\ x \in (-1, 1), \tag{12}$$

$$S(0) = 0,\ D(0) = 0. \tag{13}$$

We assume that $S(T) = 0$. Let $\theta : [0, T] \times (-1, 1) \to \mathbb{C}$ be defined by

$$\theta(t, x) := e^{i\lambda_1 t}\psi(t, x),\ (t, x) \in (0, T) \times (-1, 1). \tag{14}$$

Not STLC: A quantity with a sign whatever is the control

One defines $V(t) := -i + i\int_{-1}^{1}\theta(t,x)\varphi_1(x)$. Simple computations show that

$$V(t) = \int_0^t S(\tau)V_0(\tau)d\tau + \int_0^t S(\tau)^2 V_1(\tau)d\tau \tag{15}$$

with

$$V_0(\tau) := 2i\int_{-1}^{1}\theta(\tau,x)\varphi_{1x}(x)dx,\ V_1(\tau) := -\frac{i}{2}\int_{-1}^{1}\theta_t(\tau,x)x^2\varphi_1(x)dx.$$

Standard estimates lead to

$$V_0(t) = S(t) + O(\|S\|_{L^1(0,t)} + \varepsilon\|S\|_{L^2(0,T)} + \varepsilon|S(t)|), \tag{16}$$

$$V_1(t) = O(\varepsilon). \tag{17}$$

... Hence the real part of $V(t)$ is positive for t small enough and $S \not\equiv 0$ on $[0,t]$. Typical obstruction: a quantity which should be unsigned has a sign whatever is the control. It is very classical for finite dimensional control systems.

A Burgers control system

Let us consider the following Burgers control system (introduced by S. Guerrero)

$$(18) \qquad \begin{cases} y_t - y_{xx} + y y_x = u(t),\ t \in [0,T],\ x \in [0,L], \\ y(t,0) = 0,\ y(t,L) = 0,\ t \in [0,T], \end{cases}$$

where, at time t, the state is $y(t,\cdot) \in L^2(0,L)$ and the control is $u(t) \in \mathbb{R}$.

Theorem (F. Marbach (2018))

The control system (18) *is not small-time locally controllable.*

Marbach's proof relies on scaling, power series expansions and new quadratic estimates leading to a quantity which should be unsigned has a sign whatever is the control.

Two other examples

Obstruction to STLC for

1. 1D Schrödinger equations with bilinear controls (K. Beauchard and M. Morancey (2014))
2. A nonlinear parabolic equation (K. Beauchard and F. Marbach (2020)).

(Of course there are also classical obstructions to STLC when there is a finite speed of propagation. However there are of a different nature.) In all these cases the obstruction to STLC relies on a quantity which should be unsigned has a sign whatever is the control. The difficult part it to find this quantity and prove that it has a sign.

Main novelties of our obstruction to STLC for KdV

- This is the first case dealing with boundary controls. In our case one does not know what are the iterated Lie brackets even heuristically. Let us take this opportunity to point out that, even if they are expected to not leave in the state space (see JMC (2007)), that would be very interesting to understand what are these iterated Lie brackets.
- It sounds difficult to perform the change of time-scale introduced by F. Marbach (2018) for a Burgers control system in our situation. Indeed this change will also lead to a boundary layer. However one can no longer use the maximum principle to study this boundary layer. Moreover if the change of time-scale, if justified, allows simpler computations, the advantage for not using it might be to get better or more explicit time for the obstruction to small-time local controllability.
- The linear drift term of the linearized control system is neither self-adjoint nor skew-adjoint. Moreover its eigenvalues and eigenfunctions are not explicit.

A toy example

Let us consider the finite dimensional control system

$$\dot{y}_1 = u, \quad \dot{y}_2 = y_3, \quad \dot{y}_3 = -y_2 + 2y_1 u, \tag{19}$$

where the state is $(y_1, y_2, y_3) \in \mathbb{R}^3$ and the control is $u \in \mathbb{R}$. The linearized control system of our toy control system around $(0,0) \in \mathbb{R}^3 \times \mathbb{R}$ is

$$\dot{y}_1 = u, \quad \dot{y}_2 = y_3, \quad \dot{y}_3 = -y_2, \tag{20}$$

which is clearly not controllable. An obstruction to small-time local controllability of our toy control system (19) can be obtained by pointing out that if $(y,u) : [0,T] \to \mathbb{R}^3 \times \mathbb{R}$ is a trajectory of the toy control system (19) such that $y(0) = 0$, then

$$y_2(T) = \int_0^T \cos(T-t) y_1^2(t) dt, \tag{21}$$

$$y_3(T) = y_1(T)^2 - \int_0^T \sin(T-t) y_1^2(t) dt. \tag{22}$$

Hence,

$$y_2(T) \geq 0 \text{ if } T \in [0, \pi/2] \tag{23}$$

$$y_3(T) \leqslant 0 \text{ if } T \in [0, \pi] \text{ and } y_1(T) = 0, \tag{24}$$

which both show that our toy control system is not small-time locally controllable. More precisely, using (24), is not locally controllable in time $T \in [0, \pi]$ ((23) gives only an obstruction for $T \in [0, \pi/2]$). For the toy control system one knows that it is locally controllable in a large enough time and the optimal time for local controllability is also known: this control system is locally controllable in time T if and only if $T > \pi$. Moreover, if there are higher order perturbations (with respect to the weight $(r_1, r_2, r_3) = (1, 2, 2)$ for the state and 1 for the control) one can still get an obstruction to small-time local controllability by pointing out that the two previous obstructions respectively imply the following coercivity properties

$$(25) \qquad \forall T \in (0,\pi/2),\ \exists \delta > 0 \text{ s. t. } y_2(T) \geq \delta |u|^2_{H^{-1}(0,T)},$$

$$(26) \qquad \forall T \in (0,\pi],\ \exists \delta > 0 \text{ s. t. } (y_1(T) = 0 \Rightarrow y_3(T) \leqslant -\delta |u|^2_{H^{-2}(0,T)}).$$

Note that inequality (25) does not require any condition on the control, while (26) requires that the control is such that $y_1(T) = 0$. On the other hand it is (26) which gives the largest time for the obstruction to local controllability in time T: (25) gives an obstruction for $T \in [0, \pi/2)$, while (26) gives an obstruction for $T \in [0, \pi]$, which in fact optimal as mentioned above.

Remark

The fact that our toy system is not STLC follows from a necessary condition due to H. Sussmann (1983) relying on iterated Lie brackets. See also the more general obstructions to STLC due to K. Beauchard and F. Marbach (2017). Unfortunately iterated Lie brackets are not so well understood for PDE controls, especially for boundary controls.

Our approach is inspired by the power series expansion method introduced by JMC and E. Crépeau (2004). The idea of this method is to search/understand a control u of the form

$$u = \varepsilon u_1 + \varepsilon^2 u_2 + \cdots . \tag{27}$$

The corresponding solution then formally has the form

$$y = \varepsilon y_1 + \varepsilon^2 y_2 + \cdots , \tag{28}$$

and the non-linear term yy_x can be written as

$$yy_x = \varepsilon^2 y_1 y_{1,x} + \cdots . \tag{29}$$

One then obtains the following systems ($x \in (0, L)$ and $t \in (0, T)$)

(30)
$$\begin{cases} y_{1,t}(t, x) + y_{1,x}(t, x) + y_{1,xxx}(t, x) = 0, \\ y_1(t, 0) = y_1(t, L) = 0, \\ y_{1,x}(t, L) = u_1(t), \end{cases}$$

(31)
$$\begin{cases} y_{2,t}(t, x) + y_{2,x}(t, x) + y_{2,xxx}(t, x) + y_1(t, x)y_{1,x}(t, x) = 0, \\ y_2(t, 0) = y_2(t, L) = 0, \\ y_{2,x}(t, L) = u_2(t). \end{cases}$$

Let us recall that for the local controllability in large time the idea (JMC and E. Crépeau (2004), E. Cerpa (2007) and E. Cerpa and E. Crépeau (2009) is then to find u_1 and u_2 such that, if $y_1(0,\cdot) = y_2(0,\cdot) = 0$, then $y_1(T,\cdot) = 0$ and the $L^2(0,L)$-orthogonal projection of $y_2(T)$ on $\mathcal{M}$ is a given (non-zero) element in $\mathcal{M}$. In JMC and E. Crépeau an expansion up to the order 3 is necessary since y_2 belongs to the orthogonal space of $\mathcal{M}$ in this case. The three papers rely on contradiction arguments using the structure of the KdV systems.

Here instead of using a contradiction argument, the strategy is to characterize all possible u_1 which steers 0 at time 0 to 0 at time T. This is done by taking the Fourier transform with respect to time of the solution y_1 and applying Paley-Wiener's theorem. We then prove, in the case $2k + l \neq 3\mathbb{N} \setminus \{0\}$, if the time T is sufficiently small, $y_2(T, \cdot)$ has to leave in some open half-space if $u_1 \neq 0$.

Notations

For $z \in \mathbb{C}$, let $(\lambda_j)_{1\leqslant j\leqslant 3} = \big(\lambda_j(z)\big)_{1\leqslant j\leqslant 3}$ be the three solutions repeated with the multiplicity of

$$\lambda^3 + \lambda + iz = 0. \tag{32}$$

Set

$$Q(z) := \sum_{j=1}^{3} (\lambda_{j+1} - \lambda_j) e^{\lambda_j L + \lambda_{j+1} L} = \begin{pmatrix} 1 & 1 & 1 \\ e^{\lambda_1 L} & e^{\lambda_2 L} & e^{\lambda_3 L} \\ \lambda_1 e^{\lambda_1 L} & \lambda_2 e^{\lambda_2 L} & \lambda_3 e^{\lambda_3 L} \end{pmatrix}, \tag{33}$$

$$P(z) := \sum_{j=1}^{3} \lambda_j (e^{\lambda_{j+2} L} - e^{\lambda_{j+1} L}) = \det \begin{pmatrix} 1 & 1 & 1 \\ e^{\lambda_1 L} & e^{\lambda_2 L} & e^{\lambda_3 L} \\ \lambda_1 & \lambda_2 & \lambda_3 \end{pmatrix}, \tag{34}$$

with the convention $\lambda_4 = \lambda_1$.

Characterization of the controls steering the linearized control system from 0 to 0

Proposition

Let $L > 0$, $T > 0$, and $u \in L^2(-\infty, +\infty)$. Assume that u has a compact support included in $[0, T]$, and u steers the linearized control system from 0 at the time 0 to 0 at the time T. Then $\hat{u}$ and $\hat{u}G/H$ satisfy the Paley-Wiener conditions

(35) $$\hat{u} \text{ and } \hat{u}G/H \text{ are entire functions,}$$

and

(36) $$|\hat{u}(z)| + \left|\frac{\hat{u}G(z)}{H(z)}\right| \leq Ce^{T|\Im(z)|},$$

for some positive constant C.

Some definitions

Let k, $l \in \mathbb{N} \setminus \{0\}$ be such that be such that

(37) $$L = 2\pi\sqrt{\frac{k^2 + kl + l^2}{3}}.$$

Let us define

(38) $$\eta_1 = -\frac{2\pi i}{3L}(2k + l), \quad \eta_2 = \eta_1 + \frac{2\pi i}{L}k, \quad \eta_3 = \eta_2 + \frac{2\pi i}{L}l,$$

(39) $$p = \frac{(2k + l)(k - l)(2l + k)}{3\sqrt{3}(k^2 + kl + l^2)^{3/2}},$$

(40) $$E := \frac{40\pi^3}{3L^3}(e^{\eta_1 L} - 1)ikl(k + l),$$

(41) $$\varphi(x) := \sum_{j=1}^{3}(\eta_{j+1} - \eta_j)e^{\eta_{j+2}x} \text{ for } x \in [0, L], \qquad \text{with } \eta_4 := \eta_1,$$

(42) $$\Psi(t, x) := \Re(E\varphi(x)e^{-ipt}).$$

Some properties

(43) $$(E \neq 0) \Leftrightarrow (2k + l \notin 3\mathbb{N}),$$

(44) $$(\Psi(0, \cdot) \neq 0) \Leftrightarrow (E \neq 0),$$

(45) $$\Psi_t + \Psi_x + \Psi_{xxx} = 0,$$

(46) $$\Psi(t, 0) = \Psi(t, L) = \Psi_x(t, 0) = \Psi_x(t, L) = 0.$$

Coercivity

Proposition

If $E \neq 0$, there exists $T_* > 0$ and $C > 0$ such that, for any (real) $u \in L^2(-\infty, +\infty)$ with $u(t) = 0$ for $t \notin (0, T_*)$ and $y(T_*, \cdot) = 0$ where y is the unique solution of the linearized KdV control system starting from 0 at time 0 and using the control u, we have

$$\int_0^{T_*} \int_0^L y^2(t,x)\Psi_x(t,x)dxdt \geq C\|u\|^2_{H^{-2/3}(\mathbb{R})}. \tag{47}$$

This is one of the main steps of our proof of the obstruction to small time local controllability.

Obstruction to the small-time local controllability of the power expansion up to the order 2

(48) $\quad y = \varepsilon y_1 + \varepsilon^2 y_2 + \dots,\ y_1$ being the order $1, y_2$ being the order 2,

(49) $\quad u = \varepsilon u_1 + \varepsilon^2 u_2,\ u_1$ being the order $1, u_2$ being the order 2.

We have

(50) $\quad y_{1t} + y_{1x} + y_{1xxx} = 0,\ y_1(t,0) = y_1(t,L) = 0,\ y_{1x}(t,L) = u_1(t),$

(51)

$$y_{2t} + y_{2x} + y_{2xxx} = -y_1 y_{1x},\ y_2(t,0) = y_2(t,L) = 0,\ y_{2x}(t,L) = u_2(t),$$

We require that $y_1(0,x) = y_1(T^*,x) = 0$. So u_1 steers the linearized control system from 0 to 0, as u in the previous proposition. Allpying this proposition one gets that

(52) $$\int_0^{T_*}\int_0^L y_1^2(t,x)\Psi_x(t,x)dxdt \geq C\|u\|^2_{H^{-2/3}(\mathbb{R})}.$$

However multiplying (51) by Ψ, using the equation and the boundary conditions satisfies by Ψ (see above) and integration by parts on gets that the left hand side of the previous inequality is

$$\int_0^L y_2(T^*,x)\Psi(T^*,x)dx - \int_0^L y_2(0,x)\Psi(0,x)dx. \tag{53}$$

Hence

$$\int_0^L y_2(T^*,x)\Psi(T^*,x)dx - \int_0^L y_2(0,x)\Psi(0,x)dx \geq C\|u\|^2_{H^{-2/3}(\mathbb{R})} \tag{54}$$

which gives an obstruction to the null-controllability of the order 2 if $\Psi(0,\cdot) \neq 0$, i.e. if $2k+l \notin 3\mathbb{N}$. Moreover it gives an inequality which is crucial to deal with the remaining terms.
It remains to indeed take care of the remaining terms. Not an easy task in fact. However when we are confident that something should work it is often a question of time (may be large time...) to prove it. We finally succeed to perform it.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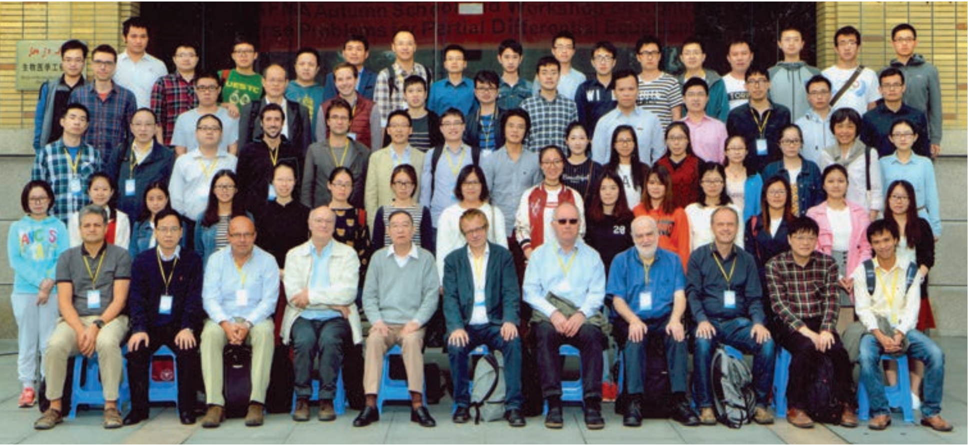

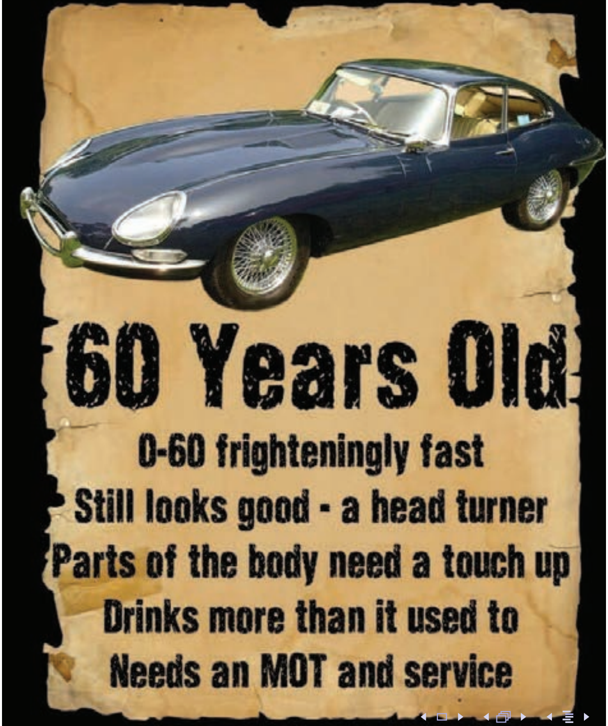
60 Years Old
0-60 frighteningly fast
Still looks good - a head turner
Parts of the body need a touch up
Drinks more than it used to
Needs an MOT and service