**PROBLEM SETTING**

Consider the small amplitude gravity water waves in a 2-D rectangular domain $\Omega$. The control $u$ on a lateral boundary is used to regulate the acceleration field of the fluid imposed by a rigid wave maker.

**Question:** How to describe the decay rate of the elevation of the free surface in a pool? How does it behave when taking the shallowness limit?

**GOVERNING EQUATIONS**

Let $\phi(t,x,y)$ be the velocity potential (for irrotational field). The motion of a inviscid, incompressible and irrotational fluid is described by the free surface Bernoulli equations $[2]$:

$$\Delta \phi = 0 \quad \text{in} \ \Omega \quad \text{(1)}$$

and $(P_{atm})$ (the constant atmospheric pressure)

$$\partial_t \phi + \frac{1}{2} |\nabla \phi|^2 + gy = -\frac{P - P_{atm}}{\rho} \quad \text{in} \ \Omega \quad \text{(2)}$$

The continuity of the pressure across the interface (without surface tension)

$$P = P_{atm} \quad \text{on} \ \Gamma_s \quad \text{(3)}$$

Based on the small-amplitude assumption (drop nonlinear terms) we obtain from (2) that

$$\partial_t \phi + g \zeta = 0 \quad \text{on} \ \Gamma_s \quad \text{(4)}$$

Boundary equations ($\mathbb{n}$: the outer normal):

- (kinematic condition) $\partial_t \phi = \partial_t \zeta$ with $\mathbb{n} = (-\nabla \zeta)/\|\nabla \zeta\|$ on $\Gamma_s$, i.e.

- (imposed boundary maker) $\partial_t \phi = v(t,y)$ on $\Gamma_1$, i.e.

- (imposing condition) $\partial_t \phi = 0$ on $\Gamma_2 \cup \Gamma_f$.

- Difficulties: non-smooth domain, mixed boundary condition

**MAIN RESULTS** [3][4]

1. Let $h \in L^2_{loc}([0,\infty))$. For every $a \in L^2([0,\infty))$, there exists a well-posed linear control system $(A,B)$ with the state $z = \dot{z} \in H^1_\omega \times H$ and input $u \in \mathcal{C}$, s.t. $z$ solves

$$\begin{align*}
\dot{z}(t) &= A z(t) + B u(t) \\
\dot{z}(0) &= z_0
\end{align*}$$

2. The well-posed linear control system $(T, \Phi)$ is strongly stabilizable, but it fails the exponential stabilizability $(T, \Phi)$ is USD if

$$\inf_{k \in \mathbb{N}, k \geq 0} \frac{1}{k} \int_0^1 h(y) \cos(k(y+1))dy > 0$$

3. For any initial data $\zeta_0 \in H^1_\omega[0,\pi]$ and $\zeta_1 \in L^2_\omega[0,\pi]$, let $\zeta_t$ be the solution of the solution of the water waves and let $\zeta$ be the solution of the wave equation with Neumann boundary control. Then we have

$$\lim_{\mu \to 0} \sup_{t \in [0,T]} \|\zeta_t - \xi\|_{H^1_\omega[0,\pi]} = 0$$

**REFERENCES**


